Magnetic braking: Finding the effective length over which the eddy currents form

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This experiment uses a square copper plate attached to a cart which moves along a track underneath a magnet. When the plate passes between the poles of the magnet, the field induces eddy currents in the plate. A damping force is caused by the interaction of the magnetic field with these currents that causes the cart to slow. By repeatedly cutting more and more slots in a sheet of metal, we could find the effective length over which the currents will form. For fins of width 2.0 ± 0.1 cm no magnetic braking was observed in a field of effective height $L = 4.4 \pm 0.2$ cm. The slope of the force versus velocity plot of this data was used to calculate the effective length as $L_R = 3.4 \pm 0.7$ cm. This value agrees with other similar experiments, whose value of the effective length is slightly smaller than the effective height of the magnetic field.

INTRODUCTION

While not discussed much at all in the past, the topic of magnetic braking has dramatically increased in popularity in recent years. Since 1987, the American Journal of Physics has published numerous articles on the subject. These articles describe both experiments dealing with magnetic braking, as well as the theory behind the phenomenon. This form of braking is so popular now that even Millennium Force, Cedar Point's new 92 mph roller coaster, uses a magnetic braking system as opposed to the traditional friction brakes.

Magnetic braking works because of induced currents and Lenz's law. If you attach a metal plate to the end of a pendulum and let it swing, its speed will greatly decrease when it passes between the poles of a magnet. When the plate enters the magnetic field, an electric field is induced and circulating "eddy currents" are generated.[1] These currents act to oppose the change in flux through the plate, in accordance with Lenz's Law.[2] The currents in turn dissipate some of the plate's energy, thereby reducing its velocity.

In order to work properly, the eddy currents need a place to form. It can be seen that when cutting slits in the plate, the damping force caused by the magnet decreases. When there are enough slits to break up the metal so that there is not a large enough area for the currents to form, damping does not occur and the plate swings unimpeded through the magnet. Cadwell[1] has experimentally determined this effective length over which the currents form to be slightly less than the effective vertical height of the magnetic field.

The practical uses for magnetic braking are numerous and commonly found in industry today. This phenomenon can be used to "damp unwanted nutations in satellites, to eliminate vibrations in spacecrafts, and to separate non-magnetic metals from solid waste."[3] Lamination, breaking up a solid piece of metal into thin sheets in order to prevent excessive energy loss due to the eddy currents (similar to our cutting slits in the metal sheet), is also common in motors.[4] In our experiment, we chose a model resembling a roller coaster to explore the aspects of magnetic braking.

THEORY

The subject of magnetic braking is rarely discussed in introductory physics texts. The more advanced electricity and magnetism textbooks scarcely mention it either, though they give more in depth descriptions of the phenomena causing magnetic braking. Smythe's book[5] on the subject, however, contains a chapter that goes into great depth on eddy currents. But the mathematics required to understand his numerous examples of exact situations is unbelievable. We will instead use a much simpler theory to explain magnetic braking.

Though recent articles in the American Journal of Physics have made the theory behind this phenomenon easily accessible, the specifics of each case are not exactly what we need. Most articles[3,6,7,8] describe the magnetic braking of a circular disk. One[9] even describes the basic physics demo of a magnet falling through a copper tube. But for our experiment, we are interested in the braking of a rectangular sheet moving linearly through the magnet. Cadwell[1] is a good source for explaining why this braking happens.

When the metal plate enters the magnetic field, it experiences a Lorentz force

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right), \qquad (1)$$

which effects the conduction of electrons in the metal. Here, $\vec{v}$ is the velocity vector of the charge q, and $\vec{B}$ is the magnetic field vector. The force on the electrons induces a current in the metal. Figure 1 shows these "eddy currents" in relation to the metal plate which moves perpendicular to the magnetic field.

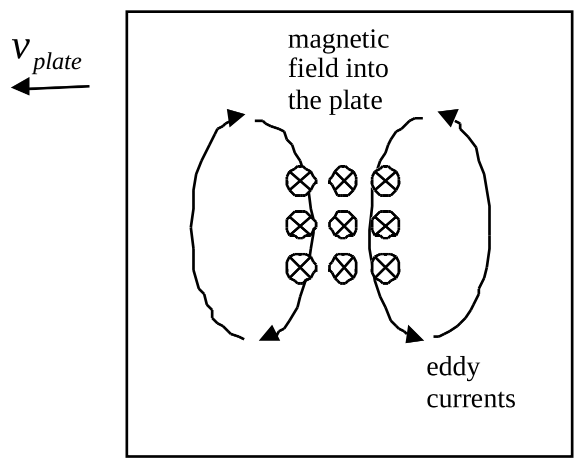

FIG. 1. Induced currents in the metal plate.

Faraday's law,

$$\varepsilon = \frac{d\Phi}{dt} = \frac{d}{dt}\int \vec{B}\ \ d\vec{A}, \qquad (2)$$

to equate this electromotive force to the velocity of the plate.[1] This involves converting the differential area to a known height times a differential width ($dA = L\,dx$), and relating the differential width to a differential time using velocity. This cancels out the integral, allowing us to write the electromotive force as

$$\varepsilon = BLv. \qquad (3)$$

Besides inducing the eddy currents in the metal plate, the magnet exerts a force on the currents inside its field. This is the retarding force associated with the braking:[1]

$$\vec{F} = I\vec{L} \times \vec{B} = ILB, \qquad (4)$$

where I is the current and L is the same vertical height of the effective magnetic field as before. The reason this and not simply the height of the magnetic poles is used is because of the fringe effects that exist outside the area directly between the two poles of the magnet. This length is determined to be the full width at half height of the Gaussian magnetic field. The simplification in (4) can be made because the length L is perpendicular to the magnetic field.

Cadwell[1] calculates the resistance the currents encounter inside the metal using the conductivity (σ) of the metal and the same area simplification as before:

$$R = \frac{L_R}{\sigma A} = \frac{L_R}{\sigma cx}, \qquad (5)$$

where L_R is the effective length over which the currents will form and c is the thickness of the metal plate.

Ohm's law lets us write a current in terms of the voltage and resistance associated with it. Using Eqs. (3) and (5), the magnitude of the eddy currents can be written as

$$I = \frac{\varepsilon}{R} = \frac{\sigma cxBL}{L_R}v. \qquad (6)$$

This allows us to rewrite the force in Eq. (4) in terms of an unknown (L_R), measurable constants, and a varying parameter (velocity):

$$F = \frac{\sigma cxB^2L^2}{L_R}v. \qquad (7)$$

However, the magnetic field strength B varies with x. To eliminate this dependence, Cadwell[1] performs a simple rectangular approximation of the sum over all positions. The result is the equation we will plot and use to calculate the effective length L_R:

$$F = ma = \frac{\sigma cL'B_{eff}^2L^2}{L_R}v, \qquad (8)$$

where

$$L' = \sum_i^n \Delta x_i \qquad (9)$$

and

$$B_{eff} = \frac{B_{max}}{2}. \qquad (10)$$

EXPERIMENT

The magnetic braking was accomplished by running a 16.5 cm x 16.5 cm x 1 mm copper plate between the poles of a large magnet. Many bars and clamps were attached to the table top in order to suspend the magnet so the cart could pass properly underneath it. A PASCO track was set up on the table. A small metal post was screwed into the top of a PASCO Dynamics Cart (ME-9430) in order to attach the metal plate. A metal hand clamp with rubber ends was used to hold the sheet of metal to the pole. This was difficult to do because the clamp used did not provide the sturdiest support, but we needed something temporary because the metal sheet would be continually removed and replaced as slits were cut in it. Figure 2 shows the experimental setup.

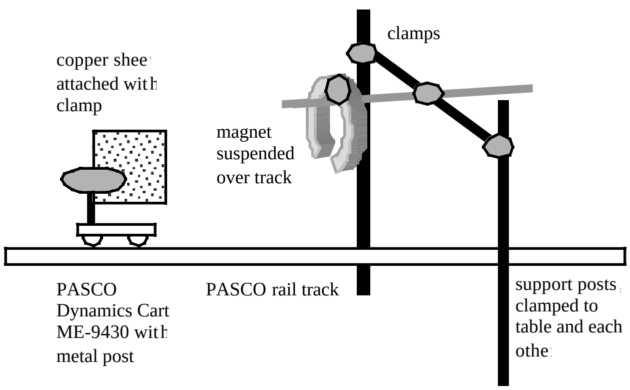

FIG. 2. Experimental setup.

Initially, data was to be taken using a PASCO Motion Sensor and Data Studio software, yielding 10 points per second. Instead, we used a video camera to tape the experiment which gave us 30 points per second. The footage was imported onto a PowerMacintosh G3 computer using Adobe Premiere and analyzed using Video Point software.

In order to test the behavior of magnetic braking, the same test was performed while varying the number of slots in the metal plate. Several runs were done by pushing the cart down the track with different initial velocities and the data analyzed using the computer. After each trial, the plate was removed from the cart and additional slits (perpendicular to the velocity) were cut (see Figure 3).

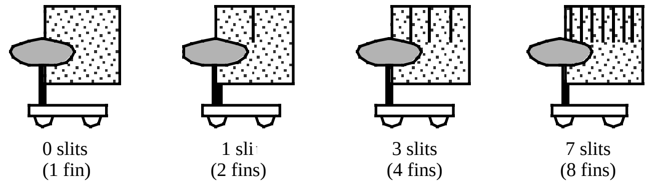

FIG. 3. Cutting slits in the metal sheet.

After the tests were complete, the various aspects of the magnetic field were measured. The PASCO CI-6520 Magnetic Field Sensor was used in conjunction with the PASCO CI-6521A Motion Sensor to coordinate field strength and position. The Dynamics Cart used in the experiment was placed directly under the magnet with the field sensor sitting on it. Data Studio software was used to coordinate the information from both sensors. With the computer recording the data, the cart was slowly moved from the center of the magnetic field to the far end. This data was used to determine the distances to the half-height ($L/2$) and zero points ($L'/2$) of the magnetic field, as well as the field strength itself (B_{max}). See Figure 4 for a combined plot of field strength versus distance.

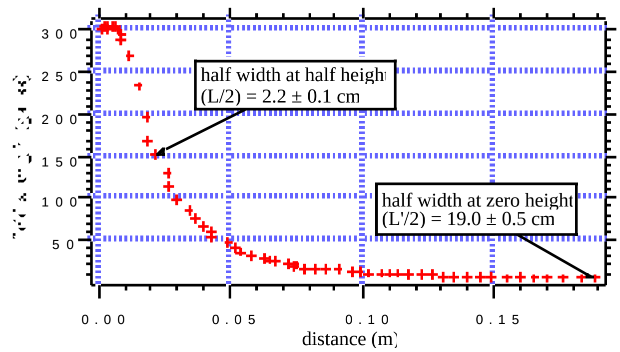

FIG. 4. Plot of field strength versus position from magnet center used to calculate the various values associated with the magnetic field.

ANALYSIS AND INTERPRETATION

When imported into Igor, the position versus time data looked perfect (see Figure 5). It showed a smooth transition from initial position through the deceleration due to the magnet and out of it again.

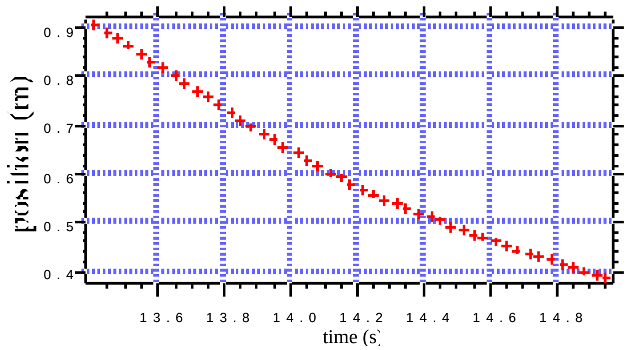

FIG. 5. Position plot showing the cart as it passes through the magnet (t=13.92 s to t=14.38 s).

From this position and time data, Excel was used to calculate the velocity at each point by finding the change in position since the previous point and dividing it by the change in time. Unlike the position data, the velocity data was a bit noisy. To fix this, Igor's built-in smoothing function was used. The results (see Figure 6) show much clearer velocity data. It shows the cart starting at an initial velocity, then drastically slowing while under the magnet. Afterwards, it slows with a different acceleration, probably due to friction and air resistance. An interesting feature of most of the data is how the cart continued to slow for a short while after exiting the magnet with the same acceleration as it had while the metal was between the magnet's poles. It seems unlikely that the fringe magnetic field would be the sole cause of this.

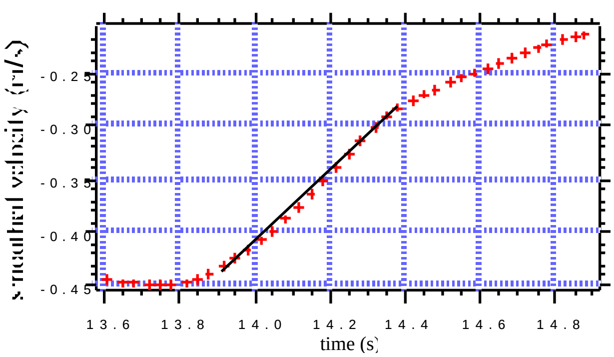

FIG. 6. Smoothed velocity data. The cart was under the magnet from t=13.92 s to t=14.38 s.

The slope of the smoothed velocity curve in the region where the metal plate was between the poles of the magnet was used to calculate the acceleration. These values would then be multiplied by the mass of the cart (m = 1.021 ± 0.001 kg) to get the magnetic damping force. The mass removed to create the slits was assumed negligible.

Figure 7 is a plot of the force and velocity data for various fin widths. The slopes are listed in Table 1. The listed error is that given by Igor from a weighted line fit for each set of data. The line representing the data from the 7-slit run is significantly out of place. This suggests that the fin width here is less than the minimum needed to sustain the eddy currents.

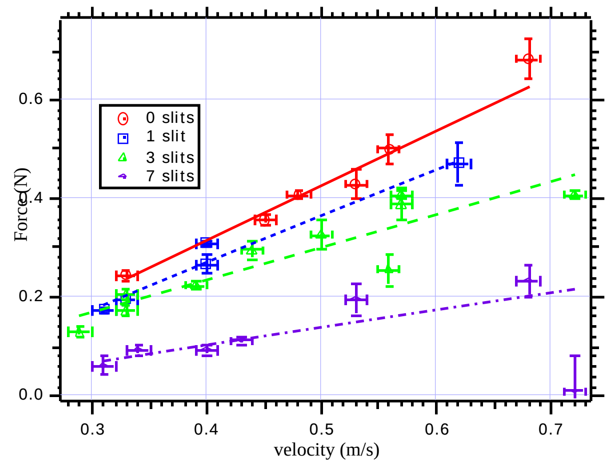

FIG. 7. Force versus velocity graph for various numbers of slits.

Slope (N/m/s)	# slits
1.10 ± 0.07	0
0.94 ± 0.09	1
0.67 ± 0.03	3
0.35 ± 0.07	7

TABLE 1. Slopes of force versus velocity graph for various numbers of slits.

From these slopes it is now possible to calculate the effective length L_R using the given value[10] of $\sigma_{Cu} = 5.92\times10^{7}$ mho/m and the measured values for the plate width ($c = (1.20 \pm 0.01)\times10^{-3}$ m), magnetic field width ($L' = 0.38 \pm 0.01$ m), effective field height ($L = 0.044 \pm 0.002$ m), and effective magnetic field strength ($B_{eff} = (1.52 \pm 0.01)\times10^{-2}$ T). A consequence of each fin width having a different slope is that we get a different value of L_R for each number of slits. This is not possible, because the effective length should be a function of the magnet, not of what goes through it. A theory about this anomaly is proposed later. Table 2 lists the effective lengths for each number of slits.

L_R (x 10^{-2} m)	# slits
1.1 ± 0.1	0
1.3 ± 0.2	1
1.8 ± 0.2	3
3.4 ± 0.7	7

TABLE 2. Effective lengths for various numbers of slits.

Now that we know the size of the eddy currents, we can investigate their magnitudes using (6). For the range of velocities and effective lengths calculated, the magnitudes of the eddy currents are on the order of 10^3 A. While this seems very high, the resistance encountered (calculated from (5)) is only on the order of 10^{-5} Ω, which puts the power consumption at around 10 W.

The biggest (and ignored) cause of error in this experiment is torque. Cadwell[1] discusses how the magnetic field should be situated so it acts on the center of mass of the apparatus passing through it. If this is not the case, besides inducing the eddy currents, the magnetic field will cause a torque on the cart about its center of mass. Due to the initial experimental setup, the magnet had to be far enough away from the cart so the motion detector could see past it, making the alignment of the center of mass and field impossible. When the switch to the video camera was made, the magnet was not lowered to try and act on the center of mass. While probably not affecting the final data that much, it is still something that could be improved in future experiments.

Looking at Figure 7, one will notice that for the two initial velocities greater than 0.70 m/s, the force observed is significantly less that what would be expected from the line fit. Cadwell[1] also observed this phenomenon. Above a certain critical velocity that probably depends on the size and magnitude of the magnetic field, the magnet no longer induces eddy currents effectively. Perhaps this is due to the time the metal plate spends within the poles of the magnet (only a couple tenths of a second at this speed). But one would think the electrons could move much faster than this. Cadwell took more data at higher velocities, and his data showed a linear decrease in force versus velocity much steeper than the slope where force increases with velocity. Further analysis with our experiment would probably show the same effect.

Along the lines of this anomaly, practically all the velocity plots show that the cart continues to decelerate after is has left the space immediately inside the magnet. But they do not show a preemptive deceleration before it enters the magnet. Perhaps there is a certain start-up time needed to generate the eddy currents that takes place when the plate first enters the fringes of the magnetic field. Once the currents form, the fringe magnetic field as the plate exits the magnet must keep them going, and continually damp the cart's motion, after the plate has left the strongest part of the field. This start-up time could also explain faster velocities resulting in less than expected damping forces. Again, further investigation of this behavior would be beneficial.

The most interesting aspect of the data we collected is the values for the effective length over which the eddy currents form. First, there needs to be reasoning behind the generation of different values for different fin widths. Or else a scaling factor needs to be introduced to compensate for the percentage of the total sheet width over which eddy currents can form. (Creation of the slits in general leads to a smaller braking force because the metal sheet contains more edges, where currents are less likely to form).

It is also interesting to note how the values for L_R change as the fin width changes. For 0, 1, and 3 slits, the effective length is smaller that the fin width. But for 7 slits, where it was assumed the fins were too narrow for the eddy currents to form, the effective length is larger than the fin width (see Table 2). Time did not permit a further investigation of this effect, but continued analysis would be very useful. As our data stands, comparing the fin width with the effective length for that number of slits can be used to determine whether the magnetic braking will work or not. Our results are similar to Cadwell,[1] who calculated the effective length to be slightly smaller than the effective width of the magnetic field. In our case, the effective field height was $L = 4.4 \pm 0.2$ cm while the effective length where the braking no longer worked was $L_R = 3.4 \pm 0.7$ cm.

CONCLUSION

As with any experiment, more time is needed to collect more data to further investigate the phenomena in question. While some of the data is still in question, there are a few facts that we are fairly certain of. First of all, the force caused by the magnet on the copper plate is proportional to the velocity, as the theory predicts. The slope of this plot can be used to determine the effective length over which the currents will form. For our calculations, when the calculated effective length was greater than the width of the fins on the metal plate (7 slits), magnetic braking would not occur. The plot of the data verified this, as the slope of the force versus velocity graph for the 7 slit data was much less, and was situated lower, than the line fit of the other data. Our data did agree with other published data that is similar: the effective length L_R is slightly less than the effective height of the magnetic field (L).

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